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Minimizing entropy generation in internal flows by adjusting the shape of the cross-section

Todd A. Jankowski *

Mechanical and Thermal Engineering Group (AET-1), Los Alamos National Laboratory, MS J580, Los Alamos, NM 87545, USA

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ABSTRACT

Entropy generation in fully-developed flow through a duct with heat transfer is discussed. Methods are presented to minimize entropy generation by adjusting the shape of the duct's cross-section. Choosing a different cross-sectional shape allows for control of the competing fluid flow and heat transfer irreversibilities. By controlling the competing irreversibilities, the total entropy generation rate can be minimized. Given the flow rate, heat transfer rate, available cross-sectional area, and the fluid properties, a general design correlation is presented that allows for a determination of the optimal shape of a duct.

1. Introduction

Forced convection heat transfer in a flow passage is affected by two types of losses, namely, loss associated with heat transfer through a temperature difference and loss associated with fluid friction. Entropy generation minimization has been proposed as a criterion for the design of flow passages in internal flow forced convection heat transfer configurations. Because entropy is generated by friction encountered in flowing fluids and by heat transfer through a temperature difference, a calculation of the overall entropy generation allows for an evaluation of these losses on a common scale. Moreover, because the entropy generation is a direct measure of the irreversibilities associated with heat transfer and fluid friction, the overall performance of a device containing heat transfer passages can be improved by calculating and minimizing the total entropy generation of the convective heat transfer process. Numerous studies have shown that in convective heat transfer arrangements the fluid friction and heat transfer losses are coupled, and that attempts to reduce entropy generation associated with heat transfer will increase the entropy generation associated with fluid friction, and vice versa [1,2]. This coupling between fluid flow and heat transfer irreversibilities suggests that the geometry and operating conditions can be optimized to minimize the overall entropy generation.

In many instances, the design engineer is faced with integrating coolant passages into an existing piece of equipment, where the space occupied by the coolant passage is at a premium and the available flow rates may be limited by the size of an existing or a retrofit fan or pump. In these situations, where a coolant passage must be designed so that the cross-sectional area is restricted to some value and where the flow rate through the coolant passage is dictated by the available equipment, one may ask the question: Is there an optimum cross-sectional shape (a circular cross-section, a square, a rectangle, etc.) for the coolant passage that minimizes entropy generation and allows for the best performance?

A number of studies have focused on the calculation and minimization of entropy generation in the fundamental fully-developed convective flow configuration in a duct. In most of these studies the entropy generation in a duct with a particular cross-sectional shape is calculated, and the entropy generation is minimized by adjusting the size (hydraulic diameter or cross-sectional area) of the duct. References can be found where entropy generation is calculated and minimized in ducts with various cross-sectional shapes for laminar and turbulent flow configurations, with constant heat transfer rate per unit length, with constant heat flux, or with constant wall temperature, and in flows with temperature dependent viscosity [3–9].

A few past studies have attempted to compare the entropy generation in ducts with different cross-sectional shapes and to determine the cross-sectional shape that will yield minimum entropy generation [10–12]. Sahin finds that for high Reynolds number flows where fluid friction irreversibility dominates, the optimal shape for a flow channel is the circular shape in both laminar flow with a constant wall temperature [10] and in turbulent flow with constant wall heat flux [11]. Sahin, however, comes to these conclusions after evaluating the entropy generation in a flow of water over rather limited ranges of parameters. Because of the limited parameter space investigated, some questions still remain on the subject of the optimal cross-section for convective heat transfer.

* Tel.: +1 505 667 5735; fax: +1 505 665 7740.
E-mail address: jankowski@lanl.gov

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Nomenclature

A	cross-sectional area
Bo	duty parameter
C_f	coefficient in friction factor
C_h	coefficient in Nusselt number
c_p	specific heat
D_h	hydraulic diameter
$\frac{dT}{dx}$	axial temperature gradient
f	Darcy friction factor
k	thermal conductivity
$\dot{m}$	mass flow rate
Nu	Nusselt number
P	perimeter
Pr	Prandtl number
$\dot{q}'$	heat transfer rate per unit length
R_A	area based Reynolds number
Re	Reynolds number
$\dot{S}'_{gen}$	entropy generation per unit length
T	temperature
Greek	
α	exponent in Nusselt number
β	exponent in Nusselt number
γ	exponent in friction factor
μ	viscosity
ρ	density
ϕ	irreversibility distribution ratio
χ	shape factor
Subscripts	
opt	at the optimum
min	minimum value

For instance, over what ranges of parameters is the circular cross-section best (to minimize entropy generation)? And, does a general design correlation exist to suggest the shape of an optimal cross-section for minimum entropy generation? To answer these questions, equations are developed here that allow for a determination of the optimal shape of a duct's cross-section given the available cross-sectional area, the heat transfer rate to or from the duct, the flow rate through the duct, and the fluid properties for fully-developed flow.

In Section 2, the equations for entropy generation and the equations describing the geometries that minimize entropy generation in steady-state flow through ducts are presented for both laminar and turbulent fully-developed flow with constant heat transfer rate per unit length. In Section 3, these equations are used to reproduce many of the results that can be found in the previous literature for flows through a duct of specified shape, where entropy generation is minimized by adjusting the size of the cross-section. Also in Section 3, new results are presented for flow in a duct of specified cross-sectional area, where entropy generation is minimized by adjusting the shape of the cross-section. These results are used to show under what circumstances a particular cross-sectional shape will minimize entropy generation. Throughout Section 3, a number of numerical examples are used when discussing results. Finally, conclusions are drawn in Section 4.

2. Entropy generation in steady-state flow through ducts

Consider the general internal flow configuration shown in Fig. 1. Fluid flows through a duct with a cross-sectional area A, a perimeter P, and a hydraulic diameter $D_h = 4A/P$. The shape of the cross-section is arbitrary but constant over the entire length of duct. A single-phase, incompressible and Newtonian fluid flows through the duct with a mass flow rate $\dot{m}$ at a bulk temperature T. Heat is transferred to the duct at a rate (per unit length) of $\dot{q}'$, through the duct wall to the fluid across a temperature difference ΔT. Following Bejan [2], for $\Delta T \ll T$, the entropy generation rate per unit length is given by

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 D_h^2}{4NukAT^2} + \frac{1}{2}\frac{f\dot{m}^3}{\rho^2 A^2 T D_h}, \tag{1}$$

where Nu, f, ρ, and k are the Nusselt number, the Darcy friction factor, the fluid density, and the fluid thermal conductivity, respectively.

Using the same notation as Ratts and Raut [4], the Nusselt number and friction factor for fully-developed laminar or turbulent flow are generalized as

$$Nu = C_h \mathrm{Re}^{\alpha}\mathrm{Pr}^{\beta}, \tag{2}$$
$$f = C_f \mathrm{Re}^{-\gamma}, \tag{3}$$

where $\mathrm{Re} = \dot{m}D_h/A\mu$ is the Reynolds number, Pr is the Prandtl number, and μ is the viscosity. The parameters C_h, C_f, α, β, and γ are tabulated in Table 1 for the circular cross-section and for rectangular and elliptical cross-sections with varying aspect ratios [13]. Additionally, in Table 1 the shape factor, defined as $\chi = P/D_h$ or $\chi = P^2/(4A) = 4A/D_h^2$, is shown for each cross-section. The shape factor is used throughout the following analysis and in the interpretation of results.

After substitution of Eqs. (2) and (3) into Eq. (1), the entropy generation rate is given by

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 D_h^2}{4C_h kAT^2 \mathrm{Re}^{\alpha}\mathrm{Pr}^{\beta}} + \frac{1}{2}\frac{C_f\dot{m}^3}{\rho^2 A^2 T D_h \mathrm{Re}^{\gamma}}. \tag{4}$$

2.1. Ducts with specified cross-sectional shape

First, consider the entropy generation in a duct while holding constant the flow rate, the heat transfer rate, and the fluid properties. Assume that the channel has a specified cross-sectional shape; that is, χ is specified. Entropy generation can then be minimized by choosing the optimum cross-sectional size for the duct.

For any duct with a specified shape factor, the size is determined by either the hydraulic diameter or the cross-sectional area, since these parameters are related through the shape factor $\chi = 4A/D_h^2$. Furthermore, the definition of the shape factor is used to write

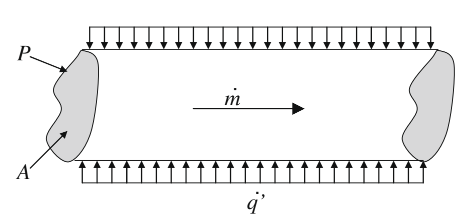

Fig. 1. The general flow and heat transfer configuration.

Table 1
Shape factors and coefficients and exponents for the Nusselt number and friction factor correlations for fully developed laminar and turbulent flow in ducts with circular, rectangular, and elliptical cross-sections.

	Aspect ratio* (a/b)	Perimeter (P)	Cross-sect. area (A)	Shape factor (χ)	α	β	γ	C_f	C_h
Laminar flow									
Circle	–	πD	$\pi D^2/4$	π	0	0	1	64	4.36
Rectangles		$2(a+b)$	ab	$2 + a/b + b/a$					
	1			4	0	0	1	56.92	3.61
	2			4.50	0	0	1	62.2	4.12
	3			5.33	0	0	1	68.36	4.79
	4			6.25	0	0	1	72.92	5.33
	6			8.17	0	0	1	78.8	6.05
	8			10.125	0	0	1	82.32	6.49
Ellipses		$(\pi/2)[2(a^2+b^2)-(a-b)^2/2]^{0.5}$	$\pi ab/4$	$(\pi/8)[3(a/b+b/a)+2]$					
	1			π	0	0	1	64	4.36
	2			3.73	0	0	1	67.28	4.56
	4			5.79	0	0	1	72.96	4.88
	8			10.36	0	0	1	76.6	5.09
	16			19.71	0	0	1	78.16	5.18
Turbulent flow									
Circle	–	πD	$\pi D^2/4$	π	0.8	0.4	0.2	0.184	0.023
Rectangles		$2(a+b)$	ab	$2 + a/b + b/a$					
	1			4	0.8	0.4	0.2	0.184	0.023
	2			4.50	0.8	0.4	0.2	0.184	0.023
	3			5.33	0.8	0.4	0.2	0.184	0.023
	4			6.25	0.8	0.4	0.2	0.184	0.023
	6			8.17	0.8	0.4	0.2	0.184	0.023
	8			10.125	0.8	0.4	0.2	0.184	0.023
Ellipses		$(\pi/2)[2(a^2+b^2)-(a-b)^2/2]^{0.5}$	$\pi ab/4$	$(\pi/8)[3(a/b+b/a)+2]$					
	1			π	0.8	0.4	0.2	0.184	0.023
	2			3.73	0.8	0.4	0.2	0.184	0.023
	4			5.79	0.8	0.4	0.2	0.184	0.023
	8			10.36	0.8	0.4	0.2	0.184	0.023
	16			19.71	0.8	0.4	0.2	0.184	0.023

* For a rectangle: [a] is the length of the longest side and [b] is the length of the shortest side; for an ellipse: [a] is the length of the major axis and [b] is the length of the minor axis.

$$\mathrm{Re} = \frac{4\dot{m}}{\chi \mu D_h}. \tag{5}$$

For a constant flow rate, fluid properties, and cross-sectional shape, Eq. (5) shows that the Reynolds number can only be varied by changing the size of the cross-section (through changes in the hydraulic diameter). Thus, the optimum cross-sectional size can be found by determining the Reynolds number that minimizes entropy generation.

Using the definitions for the shape factor and Reynolds number, and substituting into Eq. (4),

$$\dot{S}'_{gen} = \frac{\dot{q}'^2}{\chi C_h k T^2 \mathrm{Pr}^{\beta}} \mathrm{Re}^{-\alpha} + \frac{1}{128} \frac{C_f \chi^3 \mu^5}{\rho^2 T \dot{m}^2} \mathrm{Re}^{5-\gamma}. \tag{6}$$

The only parameter not held constant in the above equation is the Reynolds number. Differentiating and setting the result equal to zero, one finds that the optimum Reynolds number is given by

$$\mathrm{Re}_{opt} = \left[\frac{128\alpha}{\chi^4 (5-\gamma) C_f C_h}\right]^{\frac{1}{(5+\alpha-\gamma)}} Bo^{\frac{2}{(5+\alpha-\gamma)}} \mathrm{Pr}^{\frac{-\beta}{(5+\alpha-\gamma)}}, \tag{7}$$

where

$$Bo = \frac{\dot{q}' \rho \dot{m}}{\mu^{5/2} (kT)^{1/2}} \tag{8}$$

is the duty parameter. From the optimum Reynolds number, the optimum irreversibility distribution ratio (entropy generation due to fluid frictional losses divided by entropy generated by heat transfer through a temperature difference, or, the last term in Eq. (6) divided by the first term in Eq. (6) evaluated at Re_{opt}) is evaluated as

$$\phi_{opt} = \frac{\alpha}{5-\gamma}. \tag{9}$$

Finally, the departure of the entropy generation from the optimum is evaluated as

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = \frac{(5-\gamma)}{(5+\alpha-\gamma)} \left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{-\alpha} + \frac{\alpha}{(5+\alpha-\gamma)} \left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{5-\gamma}, \tag{10}$$

where $\dot{S}'_{gen,\min}$ is the minimum entropy generation rate evaluated when $\mathrm{Re} = \mathrm{Re}_{opt}$.

Once the shape of the cross-section (the shape factor) is chosen, the generalized results above can be used to find the optimum Reynolds number (or equivalently, the optimum size of a cross-section), the optimum irreversibility distribution ratio, and the departure of the entropy generation from the optimum, provided that the friction factor and Nusselt number correlations are known and can be described by Eqs. (2) and (3).

2.2. Ducts with a fixed cross-sectional area

Now consider a similar situation, where the entropy generation is minimized in a duct flow with constant flow rate, constant heat transfer rate per unit length, and while holding the fluid properties constant. Consider a channel with a specified cross-sectional area, A, and minimize entropy generation by choosing the optimal cross-sectional shape or shape parameter, χ, for the duct.

Once again, using the definitions of the shape factor and the Reynolds number, Eq. (4) can be arranged to read

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 A \mu^2}{4 C_h \dot{m}^2 k T^2 \mathrm{Pr}^{\beta}} \mathrm{Re}^{2-\alpha} + \frac{1}{2} \frac{C_f \dot{m}^4}{\rho^2 A^3 \mu T} \mathrm{Re}^{-(\gamma+1)}. \tag{11}$$

Furthermore, the Reynolds number is

$$Re = \sqrt{\frac{4}{\chi A}}\frac{\dot{m}}{\mu}. \tag{12}$$

Holding constant the fluid properties, the heat transfer rate, the flow rate, for a duct with a fixed cross-sectional area, and assuming that the coefficients and exponents in the Nusselt number and friction factor correlations are only weakly dependent on the shape of the cross-section, the only variable in Eq. (11) is the Reynolds number. The Reynolds number can only vary with variations in the shape of the cross-section.

To determine the optimal Reynolds number, or equivalently, the optimal shape for a cross-section, differentiate with respect to the Reynolds number and set the result equal to zero. This gives

$$\mathrm{Re}_{opt} = \left[\frac{(2-\alpha)}{2(\gamma+1)C_f C_h}\right]^{\frac{1}{(\alpha-\gamma-3)}} Bo^{\frac{2}{(\alpha-\gamma-3)}} R_A^{\frac{-8}{(\alpha-\gamma-3)}} \mathrm{Pr}^{\frac{-\beta}{(\alpha-\gamma-3)}}, \tag{13}$$

where R_A is a Reynolds number based on the area of the cross-section, defined by

$$R_A = \frac{\dot{m}}{\mu A^{1/2}}. \tag{14}$$

An optimal shape factor for a given flow configuration is evaluated from Eqs. (12) and (13) as

$$\chi_{opt} = 4\left[\frac{(2-\alpha)}{2(\gamma+1)C_f C_h}\right]^{\frac{-2}{(\alpha-\gamma-3)}} Bo^{\frac{-4}{(\alpha-\gamma-3)}} R_A^{\left[\frac{16}{(\alpha-\gamma-3)}+2\right]} \mathrm{Pr}^{\frac{2\beta}{(\alpha-\gamma-3)}}. \tag{15}$$

Finally, while holding the cross-sectional area constant, the optimum irreversibility distribution ratio and the departure of entropy generation from the minimum are given by

$$\phi_{opt} = \frac{(2-\alpha)}{(\gamma+1)} \tag{16}$$

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = \frac{(\gamma+1)}{(3+\gamma-\alpha)}\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{2-\alpha} + \frac{(2-\alpha)}{(3+\gamma-\alpha)}\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{-(\gamma+1)}. \tag{17}$$

3. Results and discussion

3.1. Ducts with specified cross-sectional shape

Bejan determined the entropy generation, the irreversibility distribution ratio, and the departure of the entropy generation from the minimum for a circular cross-section in both laminar and turbulent flow [3]. In Bejan's analysis with the circular tube, the mass flow rate, the heat transfer rate per unit length, and the fluid properties were held constant and the tube diameter was adjusted to minimize entropy generation. Using the values from Table 1 in Eqs. (7) and (9) for the circular tube carrying laminar flow, one finds that $\mathrm{Re}_{opt} = 0$ and that $\phi_{opt} = 0$. As suggested by Bejan, for a circular tube with laminar flow, the tube diameter should be large enough so that the entropy generation is dominated by the heat transfer contribution, which will result in a small value for the irreversibility distribution ratio, ϕ. As discussed above, this behavior for laminar tube flow is confirmed by the generalized expression in Eq. (9).

Using Table 1 and Eqs. (7), (9), and (10) for turbulent flow in a circular tube, the following are evaluated:

$$\mathrm{Re}_{opt} = 2.023 Bo^{0.357}\mathrm{Pr}^{-0.071} \tag{18}$$

$$\phi_{opt} = 0.167 \tag{19}$$

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = 0.857\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{-0.8} + 0.143\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{4.8}. \tag{20}$$

These equations are identical to those of Bejan. In both laminar and turbulent flow in circular tubes, the generalized results presented here reduce to expressions that are identical to the expressions derived by Bejan for circular tubes.

The generalized expressions in Eqs. (7), (9), and (10) can be used to determine the optimal size (to minimize entropy generation) of ducts with any cross-sectional shape, provided that the information contained in Table 1 are known for the flow through the particular cross-section chosen. Notice from Eq. (7) that the optimal Reynolds number is inversely proportional to the shape factor (if the exponent multiplying the bracketed term is positive, which it is for the cases considered in Table 1). Equivalently, observe from Eqs. (5) and (12) that the size (hydraulic diameter or cross-sectional area) is directly proportional to the shape factor. This result was also observed by Ratts and Raut [4] in their investigation of laminar and turbulent flow in ducts with uniform heat flux (as opposed to constant heat transfer rate per unit length studied here). As with the uniform heat flux case studied by Ratts and Raut, the equations developed here show that relatively larger cross-sectional areas and hydraulic diameters are needed for shapes with large shape factors (e.g., the large aspect ratio channels).

3.2. Ducts with a fixed cross-sectional area

For the case of internal convective heat transfer in a duct with a specified cross-sectional area, Eq. (15) is used to determine an optimal shape factor (a shape factor that minimizes entropy generation). To illustrate the determination of the optimal shape, results from Eq. (15) are presented graphically in Figs. 2 and 3.

Eq. (15) can be manipulated to give

$$\ln(Bo) = \left[\frac{5+\alpha-\gamma}{6+2\alpha-2\gamma}\right]\ln\left[\left(\frac{Bo}{R_A}\right)^2\right] + \left[\frac{\alpha-\gamma-3}{6+2\alpha-2\gamma}\right]\ln\left\{\left[\frac{(2-\alpha)}{2(\gamma+1)C_f C_h}\right]^{\frac{2}{(\alpha-\gamma-3)}}\frac{\chi_{opt}}{4\mathrm{Pr}^{2\beta/(\alpha-\gamma-3)}}\right\}. \tag{21}$$

As shown in Figs. 2 and 3, Eq. (15) results in straight lines on logarithmic scales when the parameters

$$\left(\frac{Bo}{R_A}\right)^2 = \frac{\dot{q}'^2\rho^2}{\mu^3 kT}A \quad \text{and} \quad Bo = \frac{\dot{q}'\rho}{\mu^{5/2}(kT)^{1/2}}\dot{m}, \tag{22}$$

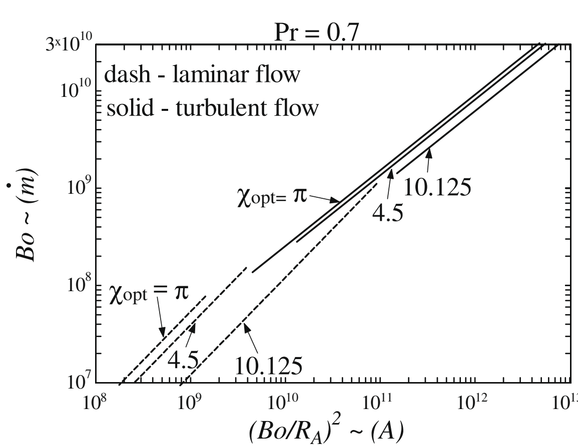

Fig. 2. The shape factor that minimizes entropy generation for flow in a duct with specified flow rate, heat transfer rate, cross-sectional area and fluid properties. The figure is generated for circular, $\chi = \pi$, and rectangular, $\chi = 4.5$ and $\chi = 10.125$, cross-sections in laminar and turbulent flow. The figure is for fluids with Pr = 0.7.

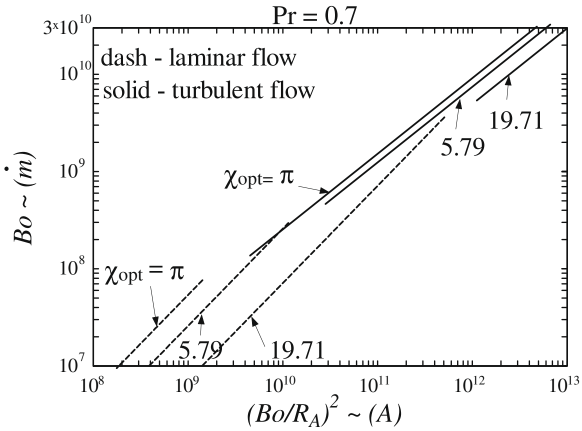

Fig. 3. The shape factor that minimizes entropy generation for flow in a duct with specified flow rate, heat transfer rate, cross-sectional area and fluid properties. The figure is generated for circular, $\chi = \pi$, and elliptical, $\chi = 5.79$ and $\chi = 19.71$, cross-sections in laminar and turbulent flow. The figure is for fluids with Pr = 0.7.

are plotted on the abscissa and ordinate, respectively. Eq. (21) shows that, regardless of the shape of the cross-section, the slope of the lines on the plot will be constant and will be determined by the exponents in the Nusselt number and friction factor correlations (although the slopes for laminar and turbulent flow may be different from one another). Fig. 2 was generated using the exponents and coefficients for the Nusselt number and friction factor correlations in Table 1 for the circular tube and the rectangular cross-sections. Fig. 3 was generated using the values in Table 1 for the elliptical cross-sections. Although the lines in Figs. 2 and 3 have a constant slope regardless of the shape of the cross-section, both the shape factor and the value of the Prandtl number (Figs. 2 and 3 were generated for Pr = 0.7) of the fluid will shift the position of the line on the plane. Most importantly, if one considers a flow with a constant heat transfer rate per unit length and with constant fluid properties, the parameter on the horizontal axis scales directly with the cross-sectional area of the duct and the parameter on the vertical axis scales directly with the mass flow rate.

For a particular convective flow with a given heat transfer rate, as soon as the flow rate and cross-sectional area are chosen, Figs. 2 and 3 can be used to determine the optimal shape factor. By way of example, consider a convective flow for which Pr = 0.7, $(Bo/R_A)^2 = 10^{12}$, and $Bo = 10^{10}$, and assume that we will use either a circular cross-section or a rectangular cross-section. In Fig. 2, this flow is represented by a point near the turbulent flow line of constant $\chi_{opt} = \pi$. These results would suggest that, for this particular flow, a circular cross-section (with $\chi = \pi$) is the optimal cross-section to minimize entropy generation.

Now consider the same flow, however, double the available cross-sectional area. The flow parameters are now Pr = 0.7, $(Bo/R_A)^2 = 2 \times 10^{12}$, and $Bo = 10^{10}$. Notice that by increasing only the cross-sectional area we move horizontally to the right in Fig. 2 (Eq. (22) shows that Bo remains constant after changing only the area). With an available cross-sectional area that is twice as large as in the previous example, the optimal shape factor is near $\chi_{opt} = 10.125$, suggesting that a rectangle with an aspect ratio of 8 is an optimal shape of the cross-section for the larger flow area duct rather than the circular cross-section.

When the available cross-sectional area of the flow channel is doubled, the resistance to flow in the duct is reduced, thereby reducing the entropy generation associated with fluid friction. However, Eq. (16) indicates that in both examples above, the optimal irreversibility distribution ratio is

$$\phi_{opt} = \frac{(2-\alpha)}{(\gamma+1)} = \left(\frac{2-0.8}{0.2+1}\right) = 1, \tag{23}$$

for these turbulent flows. To maintain this optimal irreversibility distribution ratio after an increase in the cross-sectional area (and the associated reduction in fluid friction), the entropy generation associated with heat transfer must also be reduced. The reduction in entropy generation associated with heat transfer is accomplished by choosing a cross-section with a larger perimeter compared to its cross-sectional area (a cross-section with a larger shape factor). Choosing a cross-sectional shape with a large perimeter increases the surface area available for heat transfer, reducing the entropy generation associated with heat transfer and restoring the balance required by Eq. (23) for minimum entropy generation.

This example is continued by now considering a flow with half the available cross-sectional area as in the original example, so that Pr = 0.7, $(Bo/R_A)^2 = 5 \times 10^{11}$, and $Bo = 10^{10}$. Referring again to Fig. 2, we find ourselves at a point to the left of the constant $\chi_{opt} = \pi$ line. From Eq. (15), one calculates for these flow parameters, $\chi_{opt} = 0.48$. Because the circular cross-section has the smallest possible shape factor with $\chi = \pi$, a circle would be used in this flow configuration to minimize entropy generation, although the true minimum could never be achieved.

Figs. 2 and 3 show that flows with small flow rates generally require channels with larger shape factors to minimize entropy generation. For convective heat transfer with a low flow rate in a channel with a large cross-sectional area (the lower right portions of Figs. 2 and 3), the contribution to the total entropy generation by fluid friction is relatively low. In this situation, the total entropy generation is dominated by entropy generation due to heat transfer. To minimize entropy generation, the shape factor can be increased by introducing a geometry with more surface area available for heat transfer.

Next, consider an adiabatic flow, for which $\dot{q}' = 0$. Circular or nearly circular cross-sections are used throughout engineered and natural systems to transport fluids while minimizing flow losses (flow resistance) in adiabatic flows [14]. In the case of an adiabatic flow, $Bo = 0$. In either laminar or turbulent flow, Eq. (15) suggests that $\chi_{opt} = 0$ for the adiabatic flow case. Again, because the smallest possible shape factor can be achieved with the circular cross-section, the entropy generation minimization analysis of Eq. (15) suggests that a circle is the most efficient cross-section for adiabatic flow.

Although Eq. (15) does reproduce the well-known result that the circular cross-section is the optimal shape for adiabatic flow through a duct, the entropy generation minimization presented here also provides a new result for ducts with forced convection heat transfer. The correlation in Eq. (15) and the examples shown above suggest that for flows dominated by heat transfer irreversibility (i.e., flows with large heat transfer rates, small flow rates, and large available cross-sectional area, or equivalently, flows with large Bo and small R_A) the circular cross-section is not ideal and large aspect ratio channels should be used to minimize entropy generation.

3.3. Sensitivity of entropy generation to using a non-optimal shape factor

The effect of using a non-optimal cross-section on entropy generation in flow through ducts with a fixed cross-sectional area and an arbitrary shape can be determined by combining Eqs. (12) and (17) to show that

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = \frac{(\gamma+1)}{(3+\gamma-\alpha)}\left(\frac{\chi}{\chi_{opt}}\right)^{\frac{-(2-\alpha)}{2}} + \frac{(2-\alpha)}{(3+\gamma-\alpha)}\left(\frac{\chi}{\chi_{opt}}\right)^{\frac{(\gamma+1)}{2}}. \tag{24}$$

Using the Nusselt number and friction factor correlations for turbulent flow, the departure of the entropy generation from the minimum is

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = \frac{1}{2}\left(\frac{\chi}{\chi_{opt}}\right)^{-0.6} + \frac{1}{2}\left(\frac{\chi}{\chi_{opt}}\right)^{0.6}. \qquad (25)$$

Results from Eq. (25) are shown graphically in Fig. 4. The figure shows that using a duct with a shape factor either ten times smaller than the optimum or ten times larger than the optimum doubles the entropy generated in the flow.

Although the entropy generation is increasing as one moves away from the optimum shape factor, by moving away from the optimum ($\chi/\chi_{opt} = 1$) in opposite directions, the entropy generation is increased by different mechanisms. From Eq. (25), the irreversibility distribution ratio is

$$\phi = \frac{\dot{S}'_{gen,\Delta P}}{\dot{S}'_{gen,\Delta T}} = \left(\frac{\chi}{\chi_{opt}}\right)^{1.2}. \qquad (26)$$

With shape factors smaller than the optimum, $\chi/\chi_{opt} < 1$ on the left side of Fig. 4, the total entropy generation is dominated by the heat transfer contribution and $\phi < 1$. To minimize entropy generation in this case, choose a cross-sectional shape with a larger shape factor and consequently more perimeter and surface area available for heat transfer. This choice of a cross-section with a larger surface area reduces the entropy generation associated with heat transfer while increasing the entropy generation associated with fluid friction. Moving toward an optimized cross-sectional shape will restore the balance between the fluid flow and heat transfer contributions and will move the operating point closer to the point of minimum entropy generation at $\chi/\chi_{opt} = 1$.

If, however, the shape factor of the duct is larger than the optimum, $\chi/\chi_{opt} > 1$ on the right side of Fig. 4, the total entropy generation is dominated by fluid friction and $\phi > 1$. Here, entropy generation is minimized by choosing a cross-section with a smaller shape factor. Because a cross-section with a smaller shape factor (such as the circle) has less perimeter for a given cross-sectional area, flow resistance and fluid friction are reduced. By reducing the shape factor and reducing fluid friction, the balance between entropy generation by fluid flow and heat transfer will once again be established and the total entropy generation will be minimized.

One more numerical example of entropy generation in an internal flow is used to illustrate the sensitivity of the entropy generation to using non-optimal cross-sectional shapes. Bejan [3] presented an example of minimum entropy generation for flow of air at atmospheric pressure and a temperature of $T = 1100$ K ($\rho = 0.32$ kg/m^3, $\mu = 4.35 \times 10^{-5}$ kg/m s, $c_p = 1158$ J/kg K, $k = 0.072$ W/m K, Pr = 0.7). Air flows at a rate of $\dot{m} = 100$ kg/h. Assuming a longitudinal temperature gradient of $dT/dx = 10$ K/m and $q' = \dot{m}c_p dT/dx$, one finds, $Bo = 2.6 \times 10^{10}$. The duct has a circular cross-section. Using Eq. (18), Bejan finds that the optimal Reynolds number is $\mathrm{Re}_{opt} = 1.1 \times 10^4$, which for a circular cross-section results in an optimal diameter of 7.2 cm and a cross-sectional area of $A = 41$ cm^2.

Now assume that this cross-sectional area is available and that the shape of the cross-section can be altered to further minimize entropy generation. Using the parameters above, $R_A = 1 \times 10^4$ and $Bo = 2.6 \times 10^{10}$. Eqs. (13) and (15) for turbulent flow show that $\mathrm{Re}_{opt} = 5.8 \times 10^3$ and $\chi_{opt} = 11.8$. Eq. (25) shows that by not adjusting the shape from the circular cross-section to the optimal cross-section with $\chi_{opt} = 11.8$, the entropy generation is

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = 1.33. \qquad (27)$$

In other words, entropy generation in the duct with the circular cross-section is 33% higher than entropy generation in the duct with the optimized cross-section occupying the same cross-sectional area.

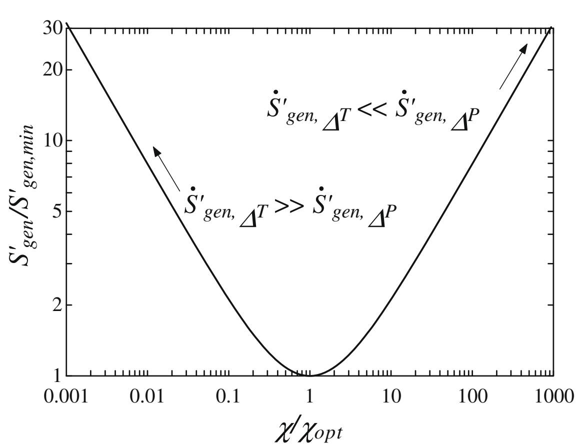

Fig. 4. Sensitivity of the total entropy generation to using a non-optimal shape factor in turbulent duct flow.

4. Conclusions

Entropy generation in fully-developed convective heat transfer has been investigated. Generalized correlations to determine the optimum cross-sectional shape of a flow passage to minimize entropy production have been presented. The equations confirm the well-known conclusion that in adiabatic flow, the circular cross-section will minimize flow resistance, which is reflected by a minimization of the entropy generation. However, in flows with heat transfer, the correlations developed suggest that the circular cross-section may not always be ideal. In situations where the heat transfer irreversibility dominates (with low flow rates, large available cross-sectional areas, and high heat transfer rates), a duct with a large wetted perimeter (for example, a rectangular channel with a large aspect ratio) will increase the surface area available for heat transfer and will minimize the overall entropy generation.

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